Self-Information transfer in maps

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The concept of transfer entropy is best understood within the context of information theory. Information is any event thatcan change the state of a system. The general idea is to optimally encode messages such that they can be transmitted more quickly. The quantify the information that can be claculated from a specific sequence of transmitted symbols[1]. Based on the measure of information transfer beweeten two variables, we define the self-inoframtion transer for time series of maps. This quantity can characterize periodc orbits.

Time-Series for Maps

We consider the time series generated by a map, such as the logistic function.

Logistic Map:

$$x_{n+1} = r x_n (1 - x_n) \quad (1)$$

We consider a time series for a given parameter value, and the one-setp delayed corresponding time serires:

The series:

$$I: \{x_0, x_1, x_2, \dots, x_{n-1}\}$$
$$J: \{x_1, x_2, x_3, \dots, x_n\} \quad (2)$$

The time serires can be transformed into symbolic dynamics:

$$x_n \le 0.5 \to 0 \quad \text{and } x_n \ge 0.5 \to 1 \quad (3)$$

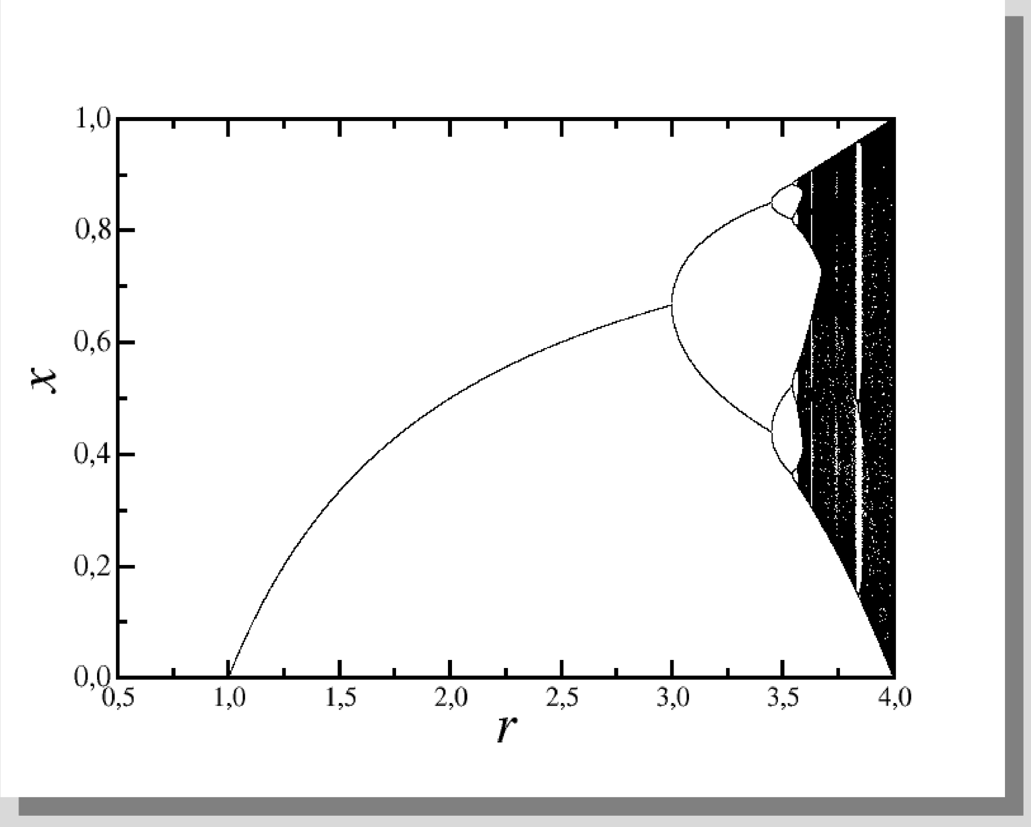

Fig 1. Bifurcation diagram for the logistic map.

Time-Series for Maps

The informatiom flow between two variables *y* and *x* can be characterized by the measure of information transfer introduced by Shreiber [1]

Transfer Information:

$$T_{y\to x} = \sum_{x_{n+1},x_n,y_n} p(x_{n+1},x_n,y_n) \log\left(\frac{p(x_{n+1},x_n,y_n)p(x_n)}{p(x_n,y_n)p(x_{n+1},x_n)} \right) \quad (4)$$

We define the Self-Information Transfer:

Self -Information Transfer:

$$T_{x_{n-1}\to x_n} = \sum_{x_{n+1},x_n,x_{n-1}} p(x_{n+1},x_n,x_{n-1}) \log\left(\frac{p(x_{n+1},x_n,x_{n-1})\,p(x_n)}{p(x_n,x_{n-1})\,p(x_{n+1},x_n)} \right) \quad (5)$$

Results

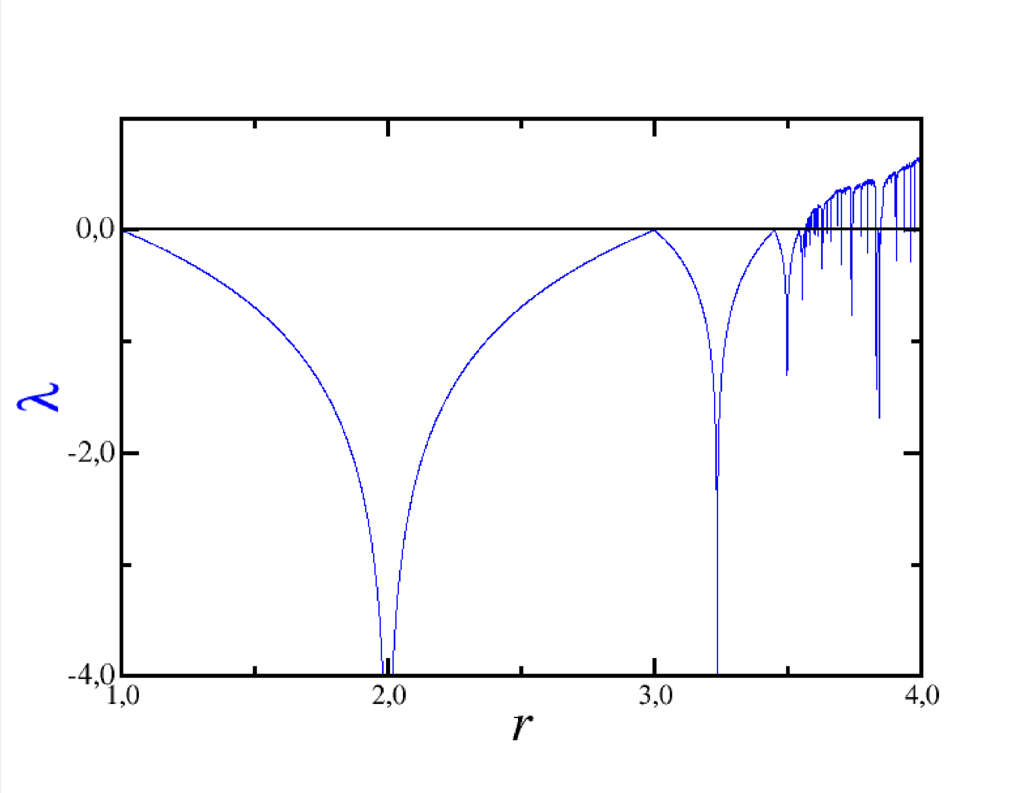

Fig 2. Lyapunov exponent for the logistic map vs. *R*.

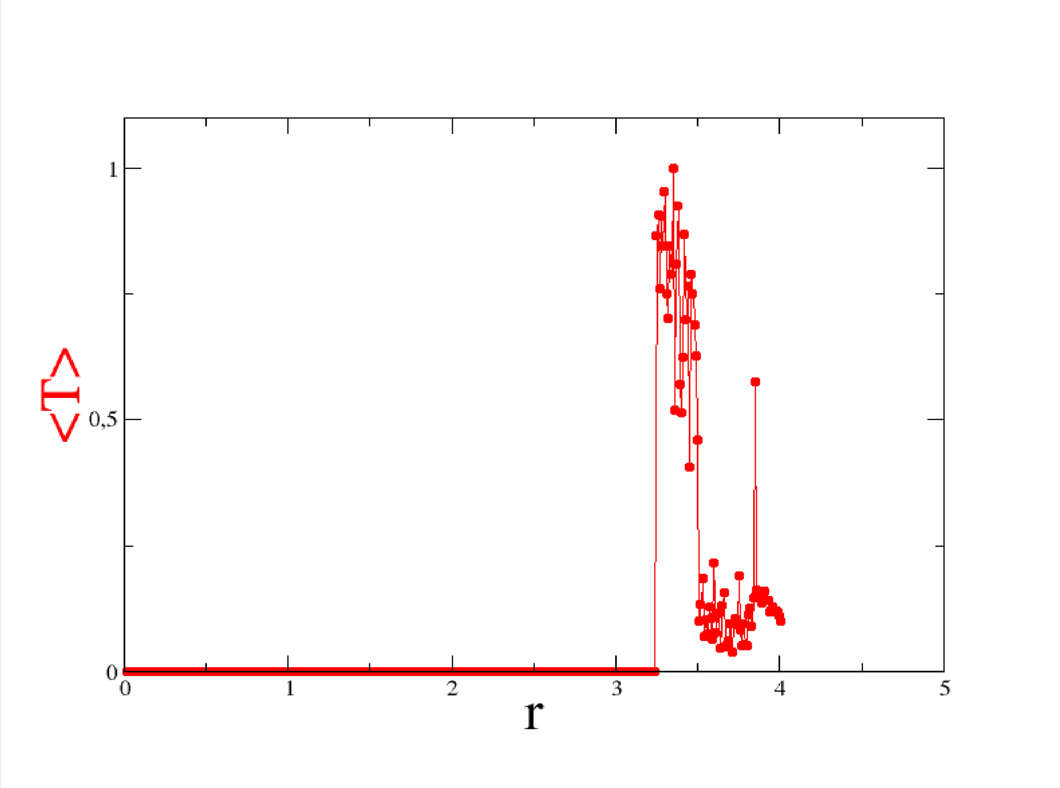

Fig 3. Averaged Self-Information Transfer $T_{x_{n-1}\to x_n}$ as a function of *r*.

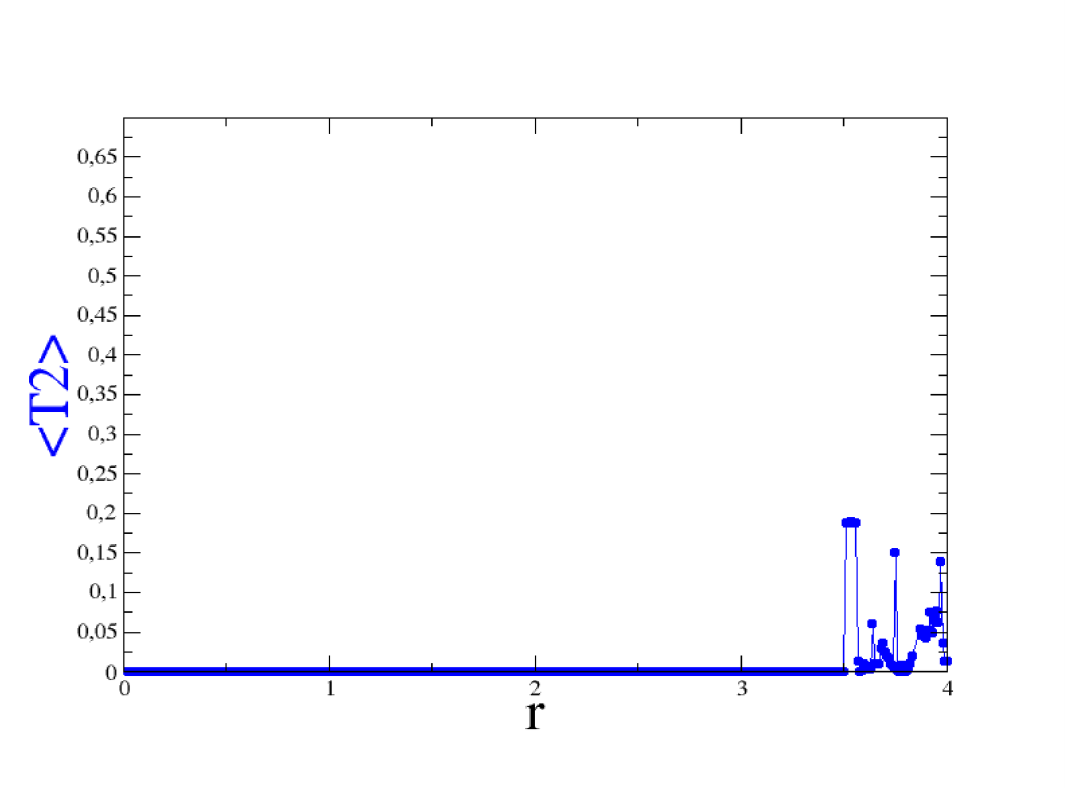

Fig 4. Averaged Self-Information Transfer $T_{x_n\to x_{n-1}}$ as a function of r

Conclusions

-Self-Information Transfer is maximum when the system is in a periodic orbit.

References

[1] Shannon, C. E. and Weaver, W. *The Mathematical Theory of Information*, University of Illinois Press, Urbana, IL.(1949)

[2] T. Schreiber. *Measuring Information Transfer.* Physical Review Letters. 85 (2) 461-464. (2000)